FORMATION OF SWIRLED JETS ESCAPING FROM ANNULAR NOZZLES

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A theoretical analysis is made of the effect of the stream parameters at the exit cross section of an annular nozzle on the flow characteristics of a swirled jet and on its form.

Data of experimental studies of swirled and unswirled jets escaping from relatively thin annular slots are presented in [1-3]. The jets propagated either in a practically unbounded space or in a half-space, i.e., in the latter case the nozzle was mounted in a wall normal to the axis of symmetry of the jet. An analysis of these experimental data, as well as of the results of studies performed by the author of the present article, shows that when the stream is sufficiently uniform along the perimeter of the nozzle a jet propagating in a space bounded by a wall takes on one of two stable forms depending on the parameters of the stream at the exit from the nozzle, namely: either the jet closes up to a certain distance from the nozzle or, spreading out, it borders on the wall and then propagates in the form of a semibounded fan jet. The main parameters determining the form of the jet are the angle of taper and the amount of swirling of the jet at the exit from the annular nozzle. Flow of the hollow annular jet type, i.e., a jet whose inner cavity is in direct contact with the surrounding medium, proved to be unstable in the case of a semi-infinite space. A hollow annular jet does occur when the jet propagates in an unbounded space, but in this case the half-aperture angle of the jet, i.e., the angle between the direction of the main stream and the axis of symmetry of the jet, cannot assume a value less than 60-65° at distances greater than one to two mean nozzle radii.

In the present article an attempt is made to determine through calculation the critical parameters of the stream at the exit from an annular nozzle for which the transition occurs from one form of a jet propagating in a semi-infinite space into the other. In addition, equations are obtained permitting the calculation of the flow in a hollow annular jet propagating in an unbounded space.

Despite the fact that the experimental data indicate the absence of flow in the form of a hollow annular jet in the case of a semi-infinite space, such a jet is theoretically possible if the condition of equilibrium in the direction perpendicular to the main stream is satisfied for it, and the half-aperture angle of the jet approaches a constant value with greater distance from the nozzle. At the same time the equilibrium of such a jet is unstable and upon the smallest disturbances the jet changes into one of the stable forms described. Thus a hollow jet satisfying the indicated requirements is a transitional form of flow between a jet which closes up and a jet which flows out along the wall. The stream parameters at the exit from an annular nozzle for which this transitional form of flow is realized will be the critical parameters.

So, in order to determine the critical stream parameters in the exit cross section of a nozzle we will examine the flow in a hollow annular jet. Let us introduce an orthogonal curvilinear coordinate system x, y, φ, where x is measured from the nozzle along the contour of a meridional cross section of the surface which is the geometrical locus of points with the maximum values of the longitudinal velocity component in cross sections of the jet, the y axis is directed along the normal to the contour, and the angle φ is measured about the axis of symmetry of the jet (Fig. 1). In the Reynolds equations for an incompressible fluid written in these coordinates with allowance for the axial symmetry and steadiness of the flow in averaged values we discard, as is generally done in studies of jets, the pulsation terms of the form $\langle u'^2 \rangle$, $\langle w'^2 \rangle$, $\langle u'w' \rangle$ and the terms due to the molecular viscosity. As a result, the system of equations describing the flow in an annular jet can be written in the form

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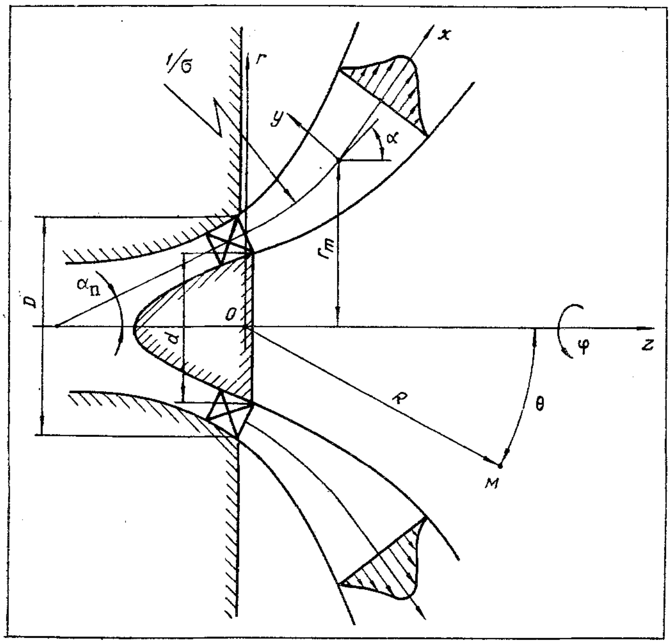

Fig. 1. Diagram of a hollow annular jet.

$$\frac{\partial}{\partial x}[(r_m + y\cos\alpha)u^2] + \frac{\partial}{\partial y}[(1-\sigma y)(r_m + y\cos\alpha)uv]$$

$$-\sigma(r_m + y\cos\alpha)uv - \sin\alpha(1-\sigma y)w^2 = -\frac{1}{\rho}(r_m + y\cos\alpha)\frac{\partial p}{\partial x}$$

$$+\frac{1}{\rho}\frac{\partial}{\partial y}[(1-\sigma y)(r_m + y\cos\alpha)\tau_u] + \sigma(r_m + y\cos\alpha)\frac{\tau_u}{\rho}, \tag{1}$$

$$\frac{\partial}{\partial x}[(r_m + y\cos\alpha)uv] + \frac{\partial}{\partial y}[(1-\sigma y)(r_m + y\cos\alpha)v^2] + \sigma(r_m + y\cos\alpha)u^2 - \cos\alpha(1-\sigma y)w^2$$

$$= -\frac{1}{\rho}(1-\sigma y)(r_m + y\cos\alpha)\frac{\partial(p + \rho\langle v'^2\rangle)}{\partial y} + \frac{1}{\rho}\frac{\partial}{\partial x}[(r_m + y\cos\alpha)\tau_u], \tag{2}$$

$$\frac{\partial}{\partial x}[(r_m + y\cos\alpha)^2uw] + \frac{\partial}{\partial y}[(1-\sigma y)(r_m + y\cos\alpha)^2vw] = \frac{1}{\rho}\cdot\frac{\partial}{\partial y}[(1-\sigma y)(r_m + y\cos\alpha)^2\tau_w], \tag{3}$$

$$\frac{\partial}{\partial x}[(r_m + y\cos\alpha)u] + \frac{\partial}{\partial y}[(1-\sigma y)(r_m + y\cos\alpha)v] = 0, \tag{4}$$

where $\tau_u = \rho\langle -u'v'\rangle$; $\tau_w = \rho\langle -w'v'\rangle$; α and σ are the half-aperture angle and the angle of curvature of the line of maximum longitudinal velocities.

We shall designate the characteristic size for the longitudinal coordinate as s and the characteristic size in the transverse direction as δ, and then because of the jet nature of the flow δ/s is a small value. We will assume that the values of r_m and $1/\sigma$ are of the same order as s. By estimating the terms in Eqs. (1)-(4) by the usual means and discarding those containing a small factor of the type δ/s, δ/r_m, or $\sigma\delta$ to more than zeroth power we obtain a system of equations describing the flow in the jet in the null approximation:

$$\frac{\partial}{\partial x}(r_m u^2) + \frac{\partial}{\partial y}(r_m uv) - w^2\sin\alpha = \frac{1}{\rho}\frac{\partial}{\partial y}(r_m\tau_u), \tag{5}$$

$$\frac{\partial}{\partial x}(r_m^2 uw) + \frac{\partial}{\partial y}(r_m^2 vw) = \frac{1}{\rho}\frac{\partial}{\partial y}(r_m^2\tau_w), \tag{6}$$

$$\frac{\partial}{\partial x}(r_m u) + \frac{\partial}{\partial y}(r_m v) = 0. \tag{7}$$

Thus, after this estimate the second equation of system (1)-(4) drops out of consideration and the pressure is taken as constant in the entire region of the jet in the null approximation.

As the boundary conditions with respect to the coordinate y for the system of equations (5)-(7) we take

$$u = w = 0 \text{ at } y = y_{1,2}, \quad \frac{\partial u}{\partial y} = 0 \text{ at } y = 0. \tag{8}$$

Suppose $u_0 = u_0(x, y)$, $w_0 = w_0(x, y)$ is the solution of system (5)-(7) with the boundary conditions (8). It is clear that the solution obtained for the region of the jet where $y \geq 0$ will coincide with the solution for the region with $y \leq 0$ except for the sign.

Let us return to Eq. (2) and retain in it the terms of first order of smallness with respect to δ/s, δ/r_m, and $\sigma\delta$, so that we will have

$$\sigma r_m u_0^2 - w_0^2 \cos\alpha = -\frac{r_m}{\rho}\frac{\partial}{\partial y}(p + \rho\langle v'^2\rangle). \tag{9}$$

We integrate Eq. (9) across the jet, and since the pressure difference at the boundaries of the jet is a value of second order of smallness we obtain

$$\sigma r_m U_0^2(x)\int_{y_1}^{y_2}\left(\frac{u_0}{U_0}\right)^2 dy - \cos\alpha W_0^2(x)\int_{y_1}^{y_2}\left(\frac{w_0}{W_0}\right)^2 dy = 0. \tag{10}$$

Equation (10) is the condition of equilibrium of the jet in the transverse direction, obtained in the first approximation, and in conjunction with the solution of system (5)-(7) it can serve to determine the dependence $r_m(x)$ in this approximation.

It is known that in swirled jets the rotational component of the velocity decreases faster than the longitudinal component. Consequently, one must take into account the terms of the next or second order of smallness in Eq. (2) to obtain the condition of equilibrium of the jet at large distances, as well as for the case of unswirled jets. But then Eqs. (1), (3), and (4) must be solved with allowance for the terms of second order of smallness.

We shall assume that the difference $(u_0/U_0) - (w_0/W_0)$ is a small value, and then it follows from a comparison of Eqs. (9) and (10) that the value $[(p - p_\infty)/\rho]$ $\langle v'^2\rangle$ is of the second order of smallness in the entire region of the jet. Thus, the allowance for terms of first order of smallness in Eqs. (1), (3), and (4) comes down to allowance for the effects of transverse and longitudinal curvature in a linear approximation. The effect of the curvature on any parameter of the jet in the region where $y \geq 0$ will differ only in sign from the corresponding value in the region with $y \leq 0$.

Retaining in Eq. (2) the terms of second order of smallness and integrating the equation obtained across the jet, we will have

$$\sigma r_m U_0^2\int_{y_1}^{y_2}\left(\frac{u_0}{U_0}\right)^2 dy - \cos\alpha W_0^2\int_{y_1}^{y_2}\left(\frac{w_0}{W_0}\right)^2 dy = r_m\left[\left(\frac{p_1 - p_\infty}{\rho} + v_1^2\right) - \left(\frac{p_2 - p_\infty}{\rho} + v_2^2\right)\right]. \tag{11}$$

The fact that corrections of the linear approximations for curvature did not enter into Eq. (11) is explained in the final analysis by the fact that upon integration across the jet these corrections, having a different sign in the regions of $y \geq 0$ and $y \leq 0$, compensated for one another.

Equation (11) in conjunction with the solution of system (5)-(7) allows one to find the dependence $r_m(x)$ in the second approximation. In this case the values of p_1 and p_2 are determined through a calculation of the flow caused by the jet in the surrounding medium, which can be considered as potential flow.

We shall use the method of integral solutions for the solution of the system of Eqs. (5)-(7). Let us examine the main section of the jet, i.e., the section where the potential core of the jet is completely diffused. We assume that the profiles of the longitudinal and rotational velocity components are similar in different cross sections of the main part of the jet, and also that they are similar to each other. Then one can write

$$u_0 = U_0(x)\cdot f(\eta), \quad w_0 = W_0(x)\cdot f(\eta), \tag{12}$$

where $\eta = \pm y/\delta_0(x)$; $\delta_0(x)$ is the half-width of the jet and $f(\eta)$ is an assigned function.

For the determination of the functions $U_0 = U_0(x)$, $W_0 = W_0(x)$, and $\delta_0 = \delta_0(x)$ three integral relations are required, the first two of which can be obtained by direct integration of Eqs. (5) and (6) across the jet. To obtain the remaining relation we transform Eq. (5), drawing on (7) and taking into account that $\partial r_m/\partial x = \sin\alpha$, to the form

$$\frac{\partial}{\partial x}(r_m u^3) + \frac{\partial}{\partial y}(r_m u^2 v) - 2w^2 u \frac{dr_m}{dx} = \frac{2}{\rho} \cdot \frac{\partial}{\partial y}(r_m u \tau_u) - \frac{2}{\rho}\tau_u \frac{\partial u}{\partial y}. \tag{13}$$

We can assign the turbulent friction τ_u, following Loitsyanskii [4], by the following expression:

$$\tau_u = \rho c^2 l^2 \sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2}\,\frac{\partial u}{\partial y}. \tag{14}$$

We take the mixing length l as constant across the jet and proportional to δ_0, and by analogy with the "new" hypothesis of Prandtl [5] we make the approximate substitution

$$\sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2} \approx \frac{\sqrt{U_0^2 + W_0^2}}{\delta_0}.$$

These assumptions and allowance for (12) allow us to reduce (4) to the form

$$\tau_u = \varkappa\rho U_0^2 \sqrt{1 + \left(\frac{W_0}{U_0}\right)^2}\,\frac{df}{d\eta}. \tag{15}$$

Let us integrate (5), (6), and (13) across the jet and apply (12) and (15). As a result we will have

$$\frac{d}{dx}(r_m U_0^2 \delta_0 a_1) - \frac{dr_m}{dx} W_0^2 \delta_0 a_1 = 0, \tag{16}$$

$$r_m^2 U_0 W_0 \delta_0 a_1 = \frac{L}{2\pi\rho}, \tag{17}$$

$$\frac{d}{dx}(r_m U_0^3 \delta_0 a_2) - 2\frac{dr_m}{dx} U_0 W_0^2 \delta_0 a_2 + 2 r_m \varkappa a_3 U_0^3 \sqrt{1 + \left(\frac{W_0}{U_0}\right)^2} = 0, \tag{18}$$

where

$$a_1 = 2\int_0^1 f^2(\eta)\,d\eta, \quad a_2 = 2\int_0^1 f^3(\eta)\,d\eta, \quad a_3 = 2\int_0^1 \left[\frac{df(\eta)}{d\eta}\right]^2 d\eta. \tag{19}$$

Let us introduce the new dependent variable

$$G = 2\pi\rho a_1 r_m U_0^2 \delta_0. \tag{20}$$

From Eq. (17) it is easy to find that

$$W_0^2 \delta_0 = \frac{L^2}{2\pi\rho a_1 r_m^3 G}. \tag{21}$$

Substituting (21) into Eq. (16) and integrating the equation obtained, we will have

$$G^2 = A - \frac{L^2}{r_m^2}. \tag{22}$$

Let us define $G(x_t) = K$ and introduce the swirl parameter

$$\Omega = \frac{L}{K r_t}, \text{ where } r_t = r_m(x_t).$$

Now determining the constant of integration in (22), we obtain

$$G = K\left[1 + \Omega^2\left(1 - \frac{r_t^2}{r_m^2}\right)\right]^{1/2}. \tag{23}$$

It follows from a comparison of Eqs. (17), (20), and (23) that

$$\left(\frac{W_0}{U_0}\right)^2=\frac{L^2}{r_m^2 G^2}=\frac{r_t^2\Omega^2}{r_m^2}\left[1+\Omega^2\left(1-\frac{r_t^2}{r_m^2}\right)\right]^{-1}. \tag{24}$$

Let us turn to Eq. (18). Using (20) and (24) this equation can be reduced to the form

$$\frac{dU_0}{dx}+\omega_1(x)U_0+\omega_2(x)U_0^3=0, \tag{25}$$

where

$$\omega_1(x)=-\frac{d\ln G}{dx},\quad \omega_2(x)=\frac{4\pi\rho a_1 a_3}{a_2}r_m\sqrt{1+\left(\frac{L}{r_m G}\right)^2}.$$

Integrating (25) with the help of the substitution $\lambda_1 = 1/U_0^2$, we will have

$$\frac{K}{U_0^2}=\left[1+\Omega^2\left(1-\frac{r_t^2}{r_m^2}\right)\right]^{-1}\left(C+\frac{8\pi\rho\varkappa a_1 a_3\sqrt{1+\Omega^2}}{a_2}\int\limits_{x_t}^{x} r_m dx\right). \tag{26}$$

We can find the integration constant C from the condition $U_0 = U_t$ at $x = x_t$.

By performing simple transformations of Eqs. (20), (24), and (26) we obtain the desired functions in the form

$$U_0=\left[1+\Omega^2\left(1-\frac{r_t^2}{r_m^2}\right)\right]^{1/2}\left(\frac{1}{U_t^2}+\frac{8\pi\rho\varkappa a_1 a_3\sqrt{1+\Omega^2}}{K a_2}\int\limits_{x_t}^{x} r_m dx\right)^{-1/2}, \tag{27}$$

$$\delta_0=\frac{K}{2\pi\rho r_m U_0^2 a_1}\left[1+\Omega^2\left(1-\frac{r_t^2}{r_m^2}\right)\right]^{1/2}, \tag{28}$$

$$W_0=\frac{r_t\Omega U_0}{r_m}\left[1+\Omega^2\left(1-\frac{r_t^2}{r_m^2}\right)\right]^{-1/2}. \tag{29}$$

Let us analyze the solution (27)-(29) at large distances from the nozzle with the condition that the half-aperture angle of the jet differs from zero, i.e., $r_m \to \infty$ as $x \to \infty$ and, in addition, $\sigma \to 0$ and $\alpha \to \alpha_\infty$ as $x \to \infty$. Then at large distances from the nozzle the coordinate x is equivalent to R, where R is the radius in the spherical coordinate system R, θ, φ (Fig. 1) while $d/dx = d/dR$. It follows from Eqs. (27)-(29) that

$$U_0\sim\frac{1}{R},\quad W_0\sim\frac{1}{R^2},\quad \delta_0=\frac{2a_3\varkappa}{a_2}R \quad \text{as } x\to\infty. \tag{30}$$

The asymptotic dependences (30) make it possible by using the results of [6, 7] to find the flow induced by the jet in the surrounding medium at large distances from the nozzle. Drawing on the indicated works, for a jet propagating in a half-space we will have

$$\begin{aligned} u_R&=-\frac{B_1}{R},\quad u_\theta=\frac{B_1(1-\cos\theta)}{R\sin\theta}\quad \text{for } \theta<\alpha_\infty;\\ u_R&=-\frac{B_2}{R},\quad u_\theta=-\frac{B_2\cos\theta}{R\sin\theta}\quad \text{for } \theta>\alpha_\infty. \end{aligned} \tag{31}$$

We can find the integration constants B_1 and B_2 from the condition that the increment in the volumetric flow rate Q through a cross section of the jet occurs due to the inflow from regions of the potential flow under consideration. Then in order to somewhat increase the accuracy of the calculation we take into account the finiteness of the width of the jet, leaving in force the result obtained in the null approximation indicating that the jet ejects the same amount of fluid from the inner and outer cavities. We can then write

$$v_1=\frac{1}{4\pi R\sin(\alpha_\infty-\gamma_\infty)}\cdot\frac{dQ}{dR},\quad v_2=\frac{1}{4\pi R\sin(\alpha_\infty+\gamma_\infty)}\cdot\frac{dQ}{dR}, \tag{32}$$

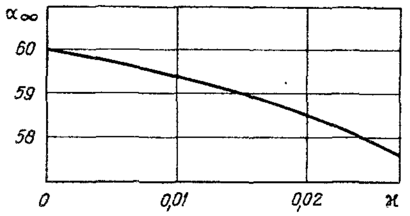

Fig. 2

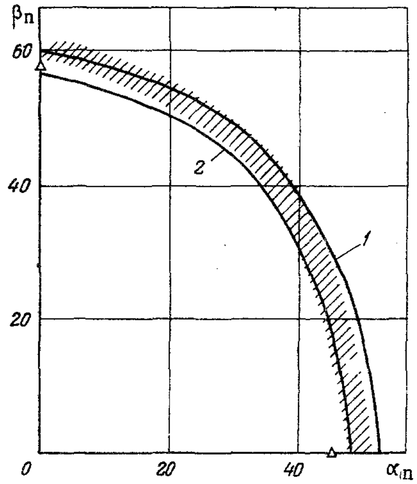

Fig. 3

Fig. 2. Dependence of half-aperture angle α_∞ on the value of the constant $\varkappa$ for a semi-infinite space (α_∞, deg; $\varkappa$ is a dimensionless value).

Fig. 3. Connection between critical values of β_n and α_n for h = 0.18 (β_n, α_n, deg): 1) $\varkappa$ = 0.011; 2) 0.015.

where γ_∞ is the angle of flare of the boundaries of the jet in the distant cross sections, with

$$\gamma_\infty = \operatorname{arctg}\frac{d\delta_0}{dR} = \operatorname{arctg}\frac{2\varkappa a_3}{a_2}. \tag{33}$$

Let us write the equations which follow from geometrical considerations:

$$v_1 = u_{\theta 1}\cos\gamma_\infty - u_{R1}\sin\gamma_\infty,\quad v_2 = u_{\theta 2}\cos\gamma_\infty + u_{R2}\sin\gamma_\infty; \tag{34}$$

$$u_1 = u_{R1}\cos\gamma_\infty + u_{\theta 1}\sin\gamma_\infty,\quad u_2 = u_{R2}\cos\gamma_\infty - u_{\theta 2}\sin\gamma_\infty. \tag{35}$$

Comparing (32) with Eqs. (34) and using (31), we find

$$B_1 = \frac{1}{4\pi(1-\cos\alpha_\infty)}\cdot\frac{dQ}{dR},\quad B_2 = \frac{1}{4\pi\cos\alpha_\infty}\cdot\frac{dQ}{dR}. \tag{36}$$

Knowing the flow induced by the jet in the surrounding medium, we can analyze the condition of equilibrium (11), which at large distances from the nozzle takes the form

$$\frac{p_1 - p_\infty}{\rho} + v_1^2 = \frac{p_2 - p_\infty}{\rho} + v_2^2. \tag{37}$$

Using the Bernoulli equation, in place of (37) we will have

$$v_1^2 - u_1^2 = v_2^2 - u_2^2. \tag{38}$$

Substituting Eqs. (31), (35), (34), and (36) into Eq. (38), we find that an equilibrium hollow jet with a constant half-aperture angle is possible if the angle α_∞ satisfies the following equation:

$$\left[\frac{\cos\alpha_\infty\sin(\alpha_\infty+\gamma_\infty)}{(\cos\gamma_\infty-\cos\alpha_\infty)\sin(\alpha_\infty-\gamma_\infty)}\right]^2 = -\frac{\cos 2\alpha_\infty}{2[1-\cos(\alpha_\infty-\gamma_\infty)]\cos(\alpha_\infty+\gamma_\infty)}. \tag{39}$$

By way of illustration, by assigning for $f(\eta)$ a dependence of the type $f(\eta) = 1-6\eta^2 + 8\eta^3 - 3\eta^4$ [7], we can find, in accordance with (19), the values of the coefficients a_1 = 0.400, a_2 = 0.232, and a_3 = 1.371; then from (33) we have $\gamma_\infty = \arctan 11.82\varkappa$. The solution of Eq. (39) for different $\varkappa$ is presented in Fig. 2.

This analysis of the solution at large distances from the nozzle thus allows one to conclude that the stream parameters at the exit from an annular nozzle will be critical in the case when the half-aperture angle $\alpha = \arcsin(dr_m/dx)$, with the dependence $r_m(x)$ calculated by Eq. (11), approaches a constant value, determined through the solution of Eq. (39), with greater distance from the nozzle.

In order to calculate the curve $r_m(x)$ by Eq. (11) one must know, in addition to the functions already determined, the laws of flow in the jet at its initial cross section and the laws of the induced flow in the region close to the nozzle.

Without dwelling in detail on the calculation of the initial cross section, we only note that by taking the profiles of the velocity components in the potential core as uniform in each cross section and using the Bernoulli equation for this region, while remaining for the rest within the framework of the assumptions adopted in the calculation of the main section, and using similar methods, one can obtain calculating functions in explicit form for the unknown parameters of the jet.

Since the effect of the pressure drop at the boundaries of the jet is a correction to the effect of the centrigual forces, this drop, and consequently the flow induced by the jet in the regions close to the nozzle, can be analyzed approximately. In particular, in the inner cavity of the jet one can assume, in analyzing the flow in the cylindrical coordinate system, r, z, φ (Fig. 1), that the profile u_z is uniform in a cross section z = const, while in the outer cavity the profile u_r is uniform at a surface r = const. These assumptions permit an easy calculation of the pressure drop at the boundaries of the jet.

By reducing the calculating functions at the initial and main cross sections to dimensionless form one can find that the determining parameters at the exit from the nozzle are the angle of taper α_n of the jet at the exit cross section (Fig. 1), the relative width $h = 4(D-d)/(D+d)\cos\alpha_n$ of the annular slot, and the swirl parameter $\Omega_n = rL_n/(D+d)K_n$ calculated from the momentum, the principal angular momentum, and the mean radius of the exit cross section of the nozzle. Here the average angle β_n between the direction of the velocity at the exit from the nozzle and the axis of symmetry of the jet is connected with the value Ω_n by the equation $\beta_n = \arctan\Omega_n$.

Curves calculated by the proposed method for two values of $\varkappa$ and reflecting the connection between the critical stream parameters at the exit from an annular nozzle for a fixed value of h = 0.18 are presented in Fig. 3. We note that the value $\varkappa = 0.011$ is taken from experimental data on flat jets and unswirled fan jets [7]. The region of values of the angle of taper α_n and the angle of swirl β_n lying below the curves obtained corresponds to a jet which closes up; the region lying above the curves corresponds to a jet which flows out along the wall. The shaded region reflects the experimental data of [2] and corresponds to the values of the angle of taper of the annular nozzle and the angle of turn of the vanes of the swirler relative to the axis of symmetry of the jet for which a jet was realized which assumed either the form of a closing-up jet or the form of a jet flowing out along the wall, depending on the external influence. As a rule, the true angle between the direction of the velocity at the exit from the nozzle and the axis of symmetry of the jet is somewhat less than the angle of turn of the vanes, and therefore a comparison of the results of the calculation with the experimental data of [2] contains a certain arbitrariness. The experimental values obtained by the author of the present article are denoted by triangles in Fig. 3. In this case the angles α_n and β_n reflect the true average direction of the stream at the exit from the nozzle for which a transition from one form of the jet to the other is possible. The satisfactory agreement of the calculated and experimental data can be noted.

In conclusion, we note that Eqs. (27)-(29) can be used to calculate the flow in a hollow annular jet propagating in an unbounded space, particularly for the calculation of a swirled fan jet.

NOTATION

x, y, φ	is the coordinate system connected with the jet;
r, z, φ	is the cylindrical coordinate system;
R, θ, φ	is the spherical coordinate system;
a_1, a_2, a_3	are the numerical coefficients;
A, B_1, B_2, C	are the integration constants;
c	is the empirical constant;
d, D	are the inner and outer diameters of exit cross section of annular nozzle;
$f(\eta)$	is the dimensionless profile of velocity components in jet;
h	is the relative width of slot;
G, K	are the momenta in jet in direction of main stream;
l	is the mixing length;
L	is the principal angular momentum in jet;
p	is the pressure;
p_∞	is the pressure in the space into which the jet escapes;

r_m	is the radius of line of maximum longitudinal velocities;
Q	is the volumetric flow rate through a cross section of the jet;
s	is the characteristic size in the longitudinal direction;
u, v, w	are the time-averaged velocity components along the x, y, and φ axes;
u', v', w'	are the pulsation velocity components;
u_r, u_z	are the velocity components along r and z axes;
u_R, u_θ	are the velocity components along R and θ axes;
U_0, W_0	are the maximum values of velocity components u_0 and w_0;
α, σ	are the half-aperture angle and curvature of line of maximum longitudinal velocities;
β	is the angle of swirl;
α_∞, γ_∞	are the half-aperture angle of jet and angle of flare of boundaries of jet in cross sections distant from the nozzle;
δ	is the characteristic size in transverse direction;
δ_0	is the half-width of jet;
η	is the dimensionless coordinate;
$\varkappa$	is the jet turbulence constant;
ρ	is the density of the fluid;
τ_u, τ_w	are the Reynolds shear stresses in directions of x and φ axes;
$\langle\ \rangle$	is the symbol of time averaging.

Subscripts and Superscripts

n refers to parameters at the exit from the nozzle;
t refers to parameters in the cross section of the transition from the initial section to the main section;
0 denotes the null approximation;
1, 2 refer to the inner and outer boundaries of the turbulent region.

LITERATURE CITED

1. D. N. Lyakhovskii, Tr. Tsentr. Aéro-Gidrodinam. Inst., No. 28 (1955).
2. H. Ulrich, Forsch. Ing.-Wesen, 26, No. 1 (1960).
3. M. Taga, Bull. Jap Soc. Mech. Eng., 13, No. 65 (1970).
4. L. G. Loitsyanskii, Prikl. Mat. Mekh., 18, No. 1 (1953).
5. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Fizmatgiz, Moscow (1960).
6. O. V. Yakovlevskii and A. N. Sekundov, Izv. Akad. Nauk SSSR, Mekh. Mashinostr., No. 3 (1964).
7. A. S. Ginevskii, Theory of Turbulent Jets and Wakes [in Russian], Mashinostroenie, Moscow (1969).